Redistribution Meets High Education Investment: Designing Taxes and Subsidies for Social Welfare

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ABSTRACT: While some papers address optimal taxation in the context of educational investment, the literature falls short in characterizing the optimal joint design of government higher education subsidies and income tax rates that simultaneously promotes redistribution and incentives for human capital accumulation. The model is solved analytically under general conditions and calibrated to assess its quantitative implications for efficiency, inequality, and welfare. Simulations show that incorporating education investment substantially alters the optimal level of income tax progressivity, with the effect depending on the responsiveness of human capital accumulation to after-tax returns. For an inequality-averse social planner, rising marginal tax rates generally emerge as optimal; yet, generous education subsidies—similar to those in several OECD countries—can justify decreasing marginal tax rates for high-income earners.
KEYWORDS: Dual Economy, Optimal income tax, Education
JEL-CODES: H21, H24, H52
DOI: https://doi.org/10.35551/PFQ_2026_2_6

Introduction

The debate over whether marginal income tax rates should decline or increase at the top remains a contentious issue. Diamond (1998) demonstrated that, under a linear utility of consumption, optimal marginal tax rates increase at the top. Saez (2001) extended this finding, reaching a similar conclusion for a non-linear utility of consumption. However, other studies argue the opposite: that optimal marginal tax rates should decline at higher income levels (e.g., Mirrlees, 1971; Tuomala, 1986; Slemrod, Yitzhaki, Mayshar, and Lundholm,

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1994). These studies suggest that, while marginal tax rates decrease at the top, progressivity can be preserved through rising average tax rates. Notably, this strand of research typically excludes education as an endogenous variable in wage determination.

A parallel body of research highlights the critical role of higher education investment in shaping income, often assuming an exogenous technology framework. This literature suggests that education-related policies, such as tax credits or subsidies for education, can influence optimal income taxation. For example, Stantcheva (2017) demonstrates this in a dynamic framework. Similarly, Jacobs (2012, 2013) argues that, with exogenous educational technology, government policies should employ regressive tools—such as subsidizing education for skilled workers at the expense of unskilled workers—to achieve optimal redistribution.

This paper contributes to desired government policy in two keyways. First, it incorporates investment in education as a central variable in income generation, addressing a significant gap in the analysis of optimal non-linear income tax rates. Second, it challenges the assumption of exogenous educational technology in higher education policy. Instead, it posits that the government can influence the education system through subsidies or regulatory measures, a feature that substantially affects the design of optimal redistributive policies.

The paper is structured as follows: Section 2 reviews relevant literature on two key topics: optimal government policy in models that account for individuals' decisions about higher education and the debate over declining versus rising marginal tax rates at the top. Section 3 presents a dual-economy model where skilled individuals invest in education while unskilled individuals remain trapped in low-wage employment. It then introduces a classical optimal non-linear income tax model with marginal taxes on skilled workers to finance transfers to the unskilled. Section 4 conducts simulations to explore the optimal policies of an inequality-averse social planner and calculates optimal income tax rates under different scenarios. Finally, Section 5 provides a summary and conclusion.

Literature Survey

Optimal tax rates with investment in education[2]

Jacobs (2012, 2013) analyzed optimal income tax rates in two contexts: optimal linear tax rates under income risk and optimal linear and non-linear tax rates in general equilibrium. In the context of risky income (Jacobs, 2012; Findeisen

2 This survey refers to optimal income taxation. Optimal rules for other taxes are highly influenced by education decisions. A good example is the inheritance tax (see Grossman and Poutvaara, 2009; and Strawczynski, 2014).

and Sachs, 2016), income taxation offers an advantage by providing insurance. By taxing both success and failure, higher income taxes reduce risk, which is a desirable feature of taxation (see Varian, 1980, for an analysis of luck as a source of income).

In the general equilibrium framework (Jacobs, 2013), which accounts for both skilled and unskilled individuals, the findings highlight the importance of subsidizing education to complement optimal private decision-making. In the non-linear context, the key result is that redistribution is desirable: unskilled individuals who do not invest in education should be taxed, while skilled individuals who do invest in education should receive subsidies.

Other studies closely related to this paper focus on short-term optimal income taxation when education investment decisions are endogenous. Notable contributions in this area include Tuomala (1986) and Maldonado (2008). These studies emphasize the role of education externalities in shaping government policy. Stantcheva (2017) adopts a dynamic approach, considering the stages of education decision-making over a lifetime. Under an exogenous education technology, her findings suggest that government influence on redistribution should come through contingent loans, enabling individuals to acquire human capital at the right timing.

In this paper, we move away from the traditional approach of treating educational technology as an exogenous factor. Instead, we explore how government policies—specifically subsidies and regulations—influence higher education outcomes (see Jacobs and Van der Ploeg, 2006).[3]

Declining or increasing optimal marginal tax rates

The debate over whether optimal non-linear tax rates should decline or increase at the top began with Mirrlees' (1971) seminal work. Mirrlees demonstrated that optimal tax rates decline at the top while average tax rates increase, preserving a progressive tax system. This conclusion was reinforced by studies in the 1980s and 1990s, including those by Atkinson (1976), Tuomala (1984), and Kanbur and Tuomala (1994). Similarly, Slemrod, Yitzhaki, Mayshar, and Lundholm (1994), using a piecewise linear tax schedule, reached a comparable conclusion.

In contrast, research from the 2000s began to challenge this view, suggesting that optimal non-linear tax rates might instead rise at the top. Diamond (1998) argued for increasing optimal marginal tax rates at higher income levels, though his findings were criticized for assuming non-existent income effects—a key determinant of optimal marginal tax rates. Saez (2001) extended this discussion

3 While there is a consensus on the need of supporting public basic education (Gradstein and Justman, 2000), that is not the case for higher education where policy changes are more controversial (Gradstein, 2019).

by incorporating a non-linear utility of consumption, providing additional support for rising optimal marginal tax rates at the top (see also Apps et al., 2014). However, this shift in perspective has not resolved the debate, which continues to depend on assumptions about utility functions and the inclusion of factors such as investment in education.

A notable evolution in the literature involves the treatment of wage distributions. Earlier studies frequently assumed a lognormal wage distribution, while contemporary research often adopts the Lognormal-Pareto distribution, which better aligns with empirical data. Under these updated assumptions, most factors—excluding income effects and the newly considered education effect—tend to support rising optimal marginal tax rates at the top.

Despite these advancements, prior studies have largely overlooked the role of endogenous education in shaping optimal marginal tax rates. Addressing this gap is one of the central contributions of this paper, which incorporates education as a critical factor in the determination of optimal taxation.

The Analytical Framework

Following non-linear optimal income tax literature, I assume that utility is both additive and separable in leisure and consumption[4]:

(1) $$u = U(C) + V(1 - L)$$

where C is consumption, 1-L is leisure and U and V are respectively the utility of consumption and the utility of leisure. U and V are differentiable, concave and non-decreasing.

I add to the standard model used for optimal income taxation a new feature: the generation of income is influenced by education decisions, which separates individuals into two groups – skilled and unskilled workers. To explain this feature let us assume that investment in higher education requires a minimal level X^*; above this level, there is a return on education at the labor market in the form of wage (w), which is obtained by individual i according to his/her learning capacity n and his/her investment in higher education X:

(2) $$w_i = ni\, X_i, \quad X_i > X^*$$

The return to education depends both on the investment in education, X, and its return, n. I use the cost of education function, h, given in the following equation:

4 The model presented here follows the analysis by Strawczynski (2025). Almost all papers that appear in the references are based on separable utility between consumption and leisure. Saez (2001) is an exception, although his simulations are also based on separable utility function.

(3) $$h(X_i) = \Gamma_0(R) + \Gamma_1(R)\frac{X_i^{1+\lambda_i}}{1+\lambda_i}$$

Where λ_i represents the cost of education, that is crucially influenced by government actions; Γ_0 and Γ_1 represent parameters that are a function of R (government regulatory burden). This specific functional form for modelling the cost of education was introduced by Lafont (1994, page 220), and it implies an increasing marginal cost of education as a function of the size of the investment. This assumption is robust, since higher stages of education are more expensive because of the higher value added to the economy.

Solving the optimum, after assuming that the cost of education can be deducted from income tax payments[5], derives in the following equation:

(4) $$X_i = \left(\frac{\boldsymbol{n_i}}{\boldsymbol{\Gamma_1}}\right)^{\boldsymbol{1/\lambda}}$$

Which implies:

(5) $$n_i = \Gamma_1 X_i^{\lambda_i}$$

i.e., at the optimum the return to education is affected by optimal investment, X, which is affected by the cost parameter, λ. The wage is:

(6) $$w_i = \widetilde{K} n_i^{\frac{1+\lambda_i}{\lambda_i}}, \quad X_i > X^*; \ \widetilde{K} = \left(\frac{1}{\Gamma_1}\right)^{1/\lambda}$$

Note that for n>1, the higher is λ, the more rigid is the demand for education for a given learning capacity. Equation 6 implies that the higher is the cost parameter λ, the lower is the wage. Thus, if the government is interested on large investment in higher education, it must assure a situation of a low λ, which will incentivate investment in higher education among the skilled.

Extending the solution shown by Dahan and Strawczynski (2012), I show in Appendix A that the following is the equation for optimal non-linear tax rates when education decision is endogenous:

(7) $$\frac{\tau}{1-\tau} = \{\varepsilon[w(X)]\} U_C\big(w(X)\big) \left[\frac{\int_{w(X)}^{\infty}\left[\frac{\gamma}{U_C(x)} - g(x)\right] \mathrm{f}(x)\mathrm{d}x}{\gamma\big(1-F(w(X))\big)}\right] \left[\frac{\big(1-F(w(X))\big)}{f\big(w(X)\big)}\right] \frac{1}{\widetilde{K} n^{\frac{1+\lambda}{\lambda}}},$$

Equation (7) is the analytical expression to be used for the calculation of optimal tax rates using the distribution of skills. This equation shows that the determination of the optimal shape of the tax schedule is given by the interaction of five effects (see Appendix B). As explained in Dahan and Strawczynski (2012), the first term in the RHS of Equation (7) is a measure of labor supply elasticity: the higher the compensated elasticity of labor the lower the optimal

5 This feature is quite usual in tax procedures in OECD countries.

marginal income tax rate. The second term, which is the marginal utility of consumption, affects the desired labor supply through its effect on labor supply. For high income levels, the marginal utility of consumption is low, and thus the incentive to work harder because of net income reduction disappears. The third effect is the "inequality aversion effect" which is increasing in w in the whole income range. The fourth effect is the "distribution effect": the higher the proportion of individuals above the income level relative to the proportion of individuals at this level, the less distortionary is the marginal tax rate, since for these individuals the marginal tax rate acts like a lump-sum tax. Thus, a higher ratio of (1-F) over f implies a higher optimal tax rate.

The new element in this equation is the last term, which implies a lower tax rate as n increases. This term gives chance for declining tax rates at the top, and thus performing simulations of optimal income tax schedule is crucial for understanding whether investment in education means declining tax rates at higher income levels.

Since the government decides about λ, I show that this feature introduces into the solution the inverse elasticity rule with respect to individual's decision about investment in education. As explained in appendix C, the last term reflects that the higher is λ the lower is the elasticity of demand for education, leading to a higher optimal marginal tax rate.

Having said that, note that the government influences the cost of education only in a partial manner, as characterized in the following equation (see appendix C):

(8) $$\lambda = \lambda^1(s) + \lambda^2 + \lambda^3$$

Where λ^1 represents the direct cost of education (matriculation), and consequently is a function of s (government's subsidy to higher education); λ^2 represents the alternative cost of working; and λ^3 represents the financial cost of loans provided by banks for financing higher education. Thus, λ (in particular, the government-influenced part) will have a crucial impact, since it determines the education's elasticity of the demand by individuals that invest above the minimal education threshold – X^*.

When choosing s, the social planner affects λ. By choosing a high s, the government would make education more accessible[6]; but at the same time, it would imply low average optimal income tax, deriving in lower tax revenue for financing re-distribution transfers. This is so because this parameter determines the rigidity of education demand λ, which is crucial for wages determination, and consequently for optimal income tax rates. The average tax rate, in turn, is crucial for financing transfers to the unskilled. In appendix C

6 While for instance it would be so also for the unskilled, in practice it is not relevant because in equilibrium the unskilled does not invest in higher education.

I show that two effects (inequality and income distribution) drive marginal tax rates up as income increases, while other two (income and educational) drive them down. In my analysis I will assume the following:

(9) $$for\ all\ j, \quad n_{Uj} = \Gamma_1 X^{\lambda_U}\ occurs\ when\ X_{Uj} < X^*$$

i.e., the desired investment in higher education by all unskilled is lower than the one needed to receive the skilled wage. Consequently, the unskilled does not invest in higher education. Concerning all skilled individuals (symbolized by i), I assume that for the different levels of n the following inequality is met:

(10) $$for\ all\ i, \quad n_{Si} = \Gamma_1 X^{\lambda_S}\ occurs\ when\ X_{Si} > X^*$$

In summary, all skilled workers invest in higher education, securing high wages proportional to their chosen level of X. In contrast, unskilled workers are constrained by their inability to invest in education, resulting in uniformly low hourly wages across this group. This outcome holds for all j.

According to my model, the government selects the higher education subsidy (s) and the regulatory mechanism (R), which together determine the accessibility of the education system. These parameters influence whether the system is open to everyone or restricted by requirements such as passing a psychometric exam, thus serving as two distinct policy variables that shape the cost of education.

Table C.1 (Appendix C) illustrates this dynamics using empirically plausible parameters, showing that the cost of education for unskilled workers is prohibitively high. This creates a significant barrier to access, complicating efforts to implement government policies aimed at addressing this fundamental issue.

Individual's Maximization of consumption and leisure

As shown by Dahan and Strawczynski (2000), it is important to consider income effects when performing the simulations. Thus, the following are the chosen functions for my simulations, which as common in the literature assume an additive utility function[7] of consumption represented by c, and leisure, represented by 1-l where l is labor:

(11) $$u_i = \mathrm{U}(c_i) + \mathrm{V}(1 - l_i)$$

7 Almost all papers that appeared in the optimal non-linear income tax literature were based on separable utility between consumption and leisure. See Mirrlees (1971), Atkinson (1973), Stern (1976), Tuomala (1984) and Diamond (1998).

The utility of consumption will cover two different cases of income effects[8]:

Normal income effect: (11)' $\mathrm{U} = \ln(c_i)$

Where income effects are represented by ; and a case with enhanced income effects:

Enhanced income effect: (11)" $\mathrm{U} = -\frac{1}{c_i}$

Where income effects are represented by $\frac{1}{c^2}$.

The leisure function is:

(12) $$\mathrm{V} = -\delta \frac{{l_i}^{1+k_i}}{1+k_i}$$

Where δ is a level parameter[9] and k can be used to calculate the labor supply elasticity, η_i. It can be shown that:

(13) $$\eta_i = \frac{1}{k_i}$$

In section 4 I calibrate the values of the parameters to fit widely accepted empirical findings in the optimal income tax literature. By looking at the analysis in Dahan and Strawczynski (2012, p.746), it is possible to understand the implications of choosing different consumption and leisure utility functions for labor supply, labor supply elasticity and the elasticity of substitution between leisure and consumption. It is clear from their analysis that the case of enhanced income effects is problematic because labor supply tends to 0 (see their Table 1 for μ>1). By contrast, for the case of normal income effects (where μ=1) and for the empirically accepted parameters as assumed in that paper labor supply converges to 0.7, which is a plausible figure for high-wage individuals. Thus, the case of normal income effects will be considered as a benchmark.

In fact, note that in the old literature (Tuomala, 1984), highest-wage skilled individuals' labor supply reaches in most scenarios 0.5, which is less accepted in the modern literature; this result is implausible in a dual economy of skilled and unskilled workers, because of the substantial impact of educational wage on labor supply.

8 Tuomala (1984) classifies papers according to the strength of income effects: normal income effects are those that derive from a logarithmic utility function as used by Mirleess (1973); enhanced income effects are the ones in which the denominator of the first derivative of consumption is elevated by 2 or more.

9 In the case of enhanced income effects labor supply tends to 0. I use as a level parameter that assures an empirically plausible labor supply.

Having said that, an undesired property of the normal income effect scenario is that labor elasticity is fixed. In my simulations I will depart from this assumption by assuming a different fixed wage elasticity for skilled and unskilled individuals (see explanation in section 4). In fact, it is well-known that high income individuals have a higher elasticity than low-income ones.

I characterize now the second stage, allowing for a simultaneous maximization by individuals and government, while each side takes the other one as given.

Government Intervention

The government collects income taxes to be used for transfers (re-distribution), and higher education subsidies. The approach of applying an income tax and transfers for re-distributing income has been widely used in the literature (Mirrlees, 1971; Saez, 2002). The following is the constraint:

$$\sum_{j=1}^{N} T\left(I_{Sj}\right) = v^{*}\mathrm{M} + \sum_{j=1}^{N} s_j \tag{14}$$

Where v is the means-tested transfer provided to the M unskilled workers and s is the education subsidy to N individuals that participate at higher education; both are financed by a non-linear income tax imposed on N skilled workers. The optimal income tax follows Apps et al. (2014,2020) in two issues: i) first, the system is valid in the short-run; i.e., individuals' decisions in education were already taken and government policy is aimed at helping low income individuals in the short run[10]; ii) second, I assume a threshold income below it there is no income tax. This assumption implies that the tax is paid by skilled individuals, while the unskilled gets transfers from the government. For simplicity, I will use in my simulations a social welfare function with a finite number of individuals:

$$\sum_{j=1}^{N} \frac{C_j^{1-\theta}}{1-\theta} \tag{15}$$

Where represents social planner's inequality aversion.

Optimal income tax schedule: simulations

To check the optimal policy, I shall simulate the decisions of social planners with different degrees of inequality aversion. We can conjecture on two

10 Since labor supply at the individual level is not observable, I assume that governments help low-income individuals according to their consumption. For a discussion on the meaning of this assumption see Strawczynski and Tirosh (2022).

extreme cases: the utilitarian (θ = 0) and the Rawlsian (θ *tends to infinity*) cases. For the utilitarian case it is quite clear that this policy maker will choose a low $\lambda_S^1(s)$ (where sub-index S represents skilled workers), since that will allow maximization of output and maximum social welfare. This is so because this type of social planner does not care about re-distribution (away from consumer's utility convexity), and consequently from his/her point of view a dual economy of the type that I have presented merely affects his educational policy choice. On the other extreme, a Rawlsian social planner would do the opposite: he/she will choose a high $\lambda_S^1(s)$, since choosing this policy will assure the possibility of imposing an optimal high average income tax, providing resources for re-distribution. This characterization hints that the interesting cases are in the middle, for different values of inequality aversion; according to values shown in the literature, I will assume that $\theta = 6$.

All simulations are based on standard assumptions, as used in other papers of optimal non-linear taxes; with respect to the parameter k in the leisure utility function V, I will assume that the compensated labor elasticity for the skilled is 0.45, following empirical findings by Gruber and Saez (2002).[11] For the unskilled I assume a fixed and lower labor elasticity (0.2).

The Distribution of earnings ability

Concerning the distribution of earnings ability, I will assume a Pareto distribution for the skilled (following Feenberg and Poterba, 1993) and a lognormal one for the unskilled (see Aitchison and Brown, 1957). Thus, I am adopting the lognormal-Pareto distribution which has been recently mentioned as a benchmark in many studies (see references in Regev and Strawczynski, 2020):

(16) *For the unskilled,* $f(n_U) = \text{lognormal}\,(\overline{n}_U, \sigma^2)$*; For the skilled,* $f(n_S) = \frac{\varepsilon \underline{n}^{\varepsilon}}{n^{1+\varepsilon}}$

Note that for the unskilled the basic wage per hour is low because of the inexistence of investment in education. Following works like Hassler and Rodriguez Mora (2000), I shall assume that the innate distribution of skills is less unequal than a distribution that is affected by social background and investment opportunities; in fact, these authors show that in equilibrium there is a correlation between the latter features and innate exogenous intelligence, which implies a more unequal distribution.

I use a typical ratio of skilled to non-skilled professions (based on Autor and Dorn, 2013) to calibrate the lognormal and Pareto means of the unskilled and

11 See their Table 9 for high income individuals. Kopczuk (2005) and Slemrod (1998) discuss other outlets for high labor supply elasticities of the skilled.

skilled hourly wage distributions, respectively. The basic income relationships between high and low-income individuals are based on Table 1.A.1.1 of OECD (2015). Note that the distribution of n is not the same as the distribution of w, and consequently I need to choose a value for that is not observable, and consequently it was not estimated in the literature. While Saez (2001) uses a value of *ε = 2* for the distribution of wages in the US, more updated estimates found that is close to 1 (Lundberg, 2024). The parallel number for European country would be 2.5. Thus, to cover a wide range, I assume that goes from 1.5 to 3.5. I perform, as well, a sensitivity analysis for the lowest return $\underline{n}$.

Government's Higher Education Subsidy

By providing a subsidy s, the government can reduce λ and generate an elastic education demand. Thus, a crucial decision is to choose the optimal value of λ, that affects $\widetilde{K}$. For simplicity I will assume that s has a direct effect on λ and R has a direct effect on $\widetilde{K}$; i.e., in the simulation, equation 6 looks as follows:

(6)' $$w_i = \widetilde{K}(R) * n_i^{\frac{1+\lambda_i(s)}{\lambda_i(s)}}, \quad X_i > X^*$$

To calibrate the cost of education I look at the participation in higher education (OECD, 2023) which arrives to 40 percent in OECD countries; accordingly, the value of λ is set to meet H(λ) = 0.4, where H is the function that translates education parameters to participation in higher education (using equation 6). As explained in Appendix Table D.2, this value is: $\lambda_S = 5$. Concerning the regulation costs, I assume that they are constant.

Simulation results

Given the optimal investment in education (equation 4), I use equation 7 to calculate the optimal non-linear tax schedule, which will determine the average tax rate in the economy. This revenue finances transfers to the unskilled, who are below the threshold income that implies a positive tax rate. Transfers are means-tested and delivered to the unskilled, whose income is below the threshold income that is valid for income tax.

While the given values for R (regulation costs) and s (education subsidy) are determined by the government, I assume that values for s are consistent with equation 8; an implicit assumption is that is optimally chosen by the government through changes in s. Table 1 summarizes the parameters choice for my simulations.

Table 1. Simulations Parameters

Parameter	Value	Based on
(Pareto parameter)	1.5-3.5	Lundberg (2024)
Wage ratio between unskilled and skilled hourly wage	0.118	Table 1.A.1.1 of OECD (2015).
Education participation H(λ)	0.4	OECD (2023)
λ (Education costs)	1-10	Strawczynski (2025)
Optimal Income Tax	t_1=0.08; t_2=0.21	Apps et al. (2014,2020)
η (Labor supply elasticity)	0.4	Gruber and Saez (2002)
θ (Inequality aversion parameter)	6	Diamond (1998); high range in Saez (2002)

Optimal Marginal tax schedule

Let me now characterize the optimal non-linear income tax rates under the different scenarios. For this purpose, I show the case of a standard inequality aversion ($\theta = 6$)) in the two inequality scenarios described above ($\alpha = 1.5$ and α 3.5.

Normal income effects

Table 2 shows the outcome of different variables for three different values of .

Table 2. Economic variables when λ_s=5: normal income effects

Scenario	ε = 1.5			ε = 3.5		
	n = 6	n = 9	n = 12	n = 6	n = 9	n = 12
Average tax rate	0.45	0.30	0.21	0.185	0.148	0.119
Max. marginal tax rate	0.67	0.67	0.67	0.67	0.67	0.67
Increasing average tax rates	YES	YES	YES	YES	YES	YES
Average wage	42.5	56.4	72.2	59.5	79.0	101.1
Ratio of skilled to unskilled wage	14.9	19.8	25.3	20.9	27.7	35.5
Skilled wage Coefficient of Variation	0.22	0.16	0.12	0.06	0.07	0.072

While average optimal tax rates rise with income in all scenarios, results show a clear difference between the case of $\varepsilon = 3.5$ relatively to ε = 2.5; in the latter case, which represents a more unequal economy (see last line which shows the skilled wage coefficient of variation), average tax rates are higher. This result was obtained in spite of the fact that the ratio of skilled to unskilled wage is lower in the unequal scenario. This result implies that an unequal education economy drives to steeper schedule of optimal marginal tax rates. Note, however, that maximal marginal tax rates are high. This result calls for further analysis, including the case of enhanced income effects which in the older literature was associated with declining optimal tax rates at the top. This task is performed in the next sub-section.

Enhanced income effects

It is worth investigating whether the result of increasing marginal tax rates at the top holds when the utility of consumption is highly non-linear ("enhanced income effects") and individuals invest in education. This scenario is analyzed in Table 3.

In this case, the average tax rate in the less equal economy ($\varepsilon = 2.5$) is higher than in the more equal economy ($\varepsilon = 3.5$). As with standard income effects, the highest average tax rate occurs at $\underline{n} = 6$, which can be explained by the wider income range, allowing for a more dispersed income distribution.

To compare normal and enhanced income effects, the next section provides a detailed analysis of optimal income tax schedules.

Table 3. Economy parameters when λ_s=5 and enhanced income effects

Scenario	$\varepsilon = 1.5$			$\varepsilon = 3.5$		
	$\underline{n} = 6$	$\underline{n} = 9$	$\underline{n} = 12$	$\underline{n} = 6$	$\underline{n} = 9$	$\underline{n} = 12$
Average tax rate	0.22	0.16	0.13	0.092	0.082	0.074
Max. marginal tax rate	0.67	0.67	0.67	0.67	0.67	0.67
Increasing average tax rates	YES	YES	YES	YES	YES	YES
Average wage	41.2	54.8	68.6	57.7	76.7	96
Ratio of skilled to unskilled wage	19.5	26.0	32.5	27.4	36.4	45.5
Skilled wage Coefficient of Variation	0.22	0.16	0.13	0.15	0.12	0.09

A comparison between normal and enhanced income effects

Figures 1a to 1c compare optimal schedules for an unequal economy; i.e., ε = 2.5.

Figure 1a. λS=5, α=1.5 and minimum n equals 6

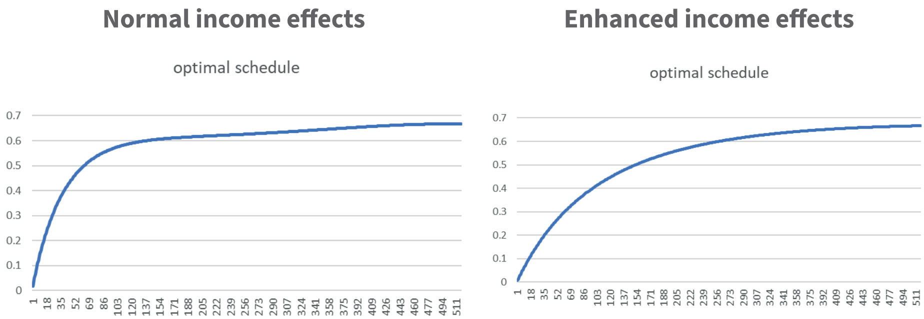


Figure 1b. λ_s=5, α=1.5 and minimum n equals 9

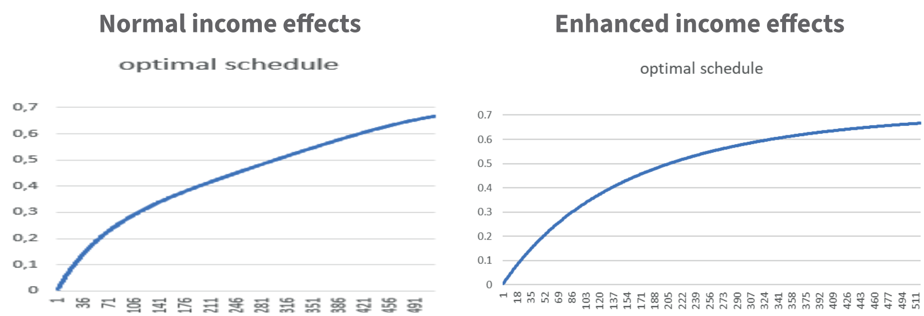
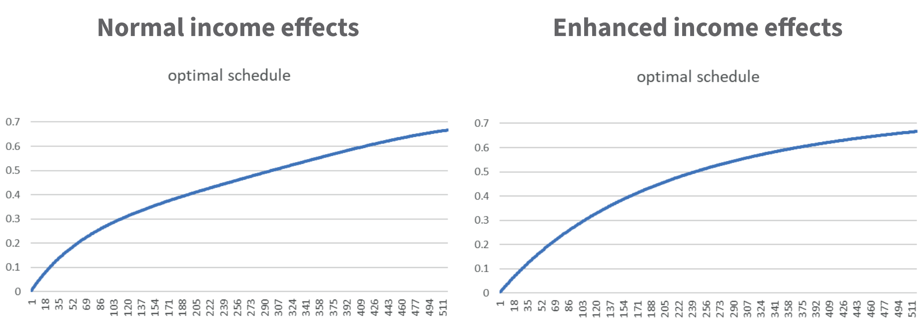


Figure 1c. λ_s=5, α=1.5 and minimum n equals 12



Figures 2a to 2c compare optimal marginal tax schedules in the more equal economy, where $\varepsilon = 3.5$.

Figure 2a. λ_s=5, ε=3.5 and minimum X equals 6

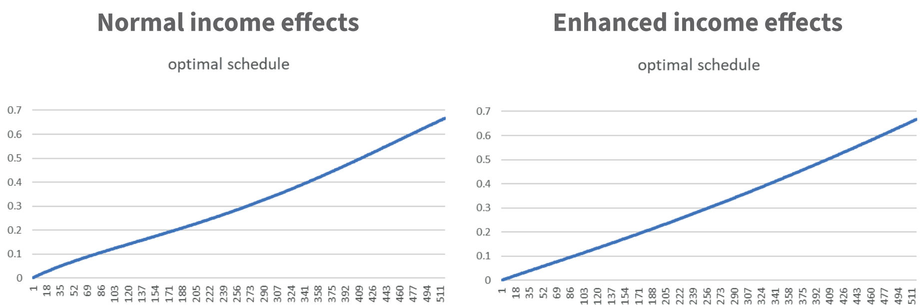


Figure 2b. λ_s=5, ε=3.5 and minimum X equals 9

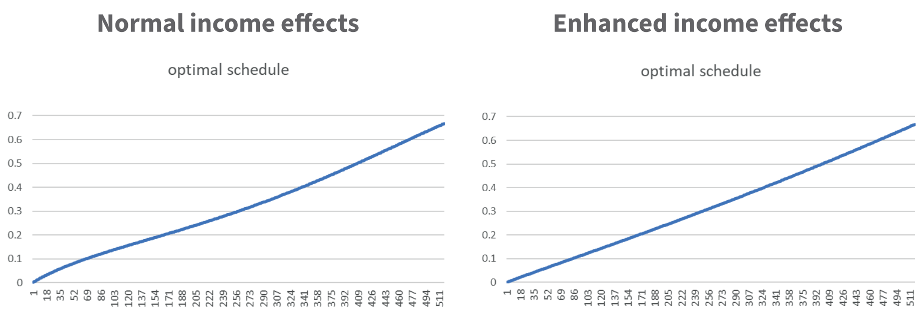
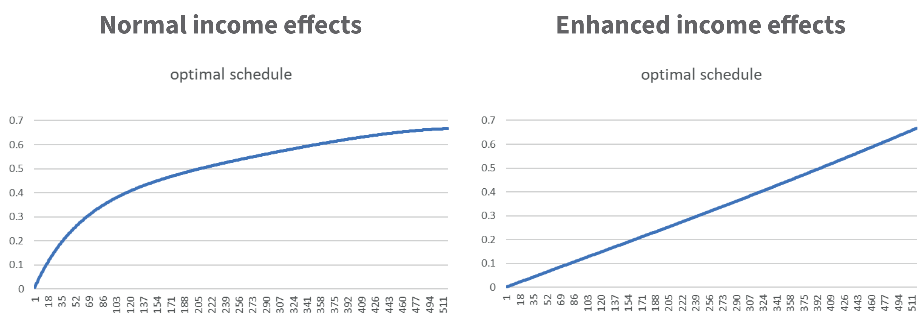


Figure 2c. λ_s=5, ε=3.5 and minimum X equals 12



By analyzing the effects outlined in Appendix B, we observe that in this framework, the income effect is amplified by the educational effect. It is worth noting, however, that the primary difference between the scenarios lies in the income effect. Consequently, the key distinction is that, under normal income effects, optimal marginal tax rates rise more sharply at the lower end of the income distribution. For higher incomes, the optimal tax rates are comparable across both scenarios. In essence, with normal income effects, tax rates increase steeply from the outset, whereas with enhanced income effects, the increase is more gradual. This distinction explains the lower average tax rates in the latter case (as shown in Table 3) compared to the former (Table 2).

The upward trajectory in marginal tax rates is largely driven by rapidly increasing inequality and the effects of income distribution, which outweigh the combined influence of income and educational effects. As a result, the most plausible outcome is rising optimal tax rates. Moreover, these rates are higher in economies with greater inequality. Overall, these simulations underscore the pivotal role of income inequality in driving the trend of rising optimal tax rates.

Higher education Government's policy: a high subsidy

By adjusting the level of higher education subsidies and the degree of regulatory stringency, the government significantly influences the level of elitism within the higher education system. In this section, I examine the impact of changes in the government's subsidy, s, which yields qualitatively similar outcomes to increasing regulatory stringency. A high subsidy (s) leads to a lower education cost (), which has two key implications: i) increased output: lower education costs contribute to higher overall output; and ii) greater elasticity in education demand: a more elastic demand for education results in lower optimal marginal tax rates. Thus, it is worth exploring whether substantial subsidies have an impact on the optimal non-linear income tax schedule.

For this purpose, I check results under normal income effects: does a higher education government subsidy imply a switch of optimal taxation schedule shape from rising to declining optimal rates? I take the middle scenario of the "equal" (ε = 3.5; $\underline{n}$ = 9) and the high scenario of the "unequal" economy (ε = 2.5; $\underline{n}$ = 12), and increase government subsidy so as to reduce education cost to the skilled (λ_S) to 3 and 1.6, respectively (from λ_S = 5, as assumed in the benchmark scenario). Table 4 and Figure 3 show the results.

Table 4. Average tax rates with higher government subsidy

Scenario	$\underline{n}$ = 9; ε = 3.5	$\underline{n}$ = 12; ε = 1.5
λ=3	0.201	0.463
λ=1.6	0.374	0.444

Figure 3a. Higher government subsidy: middle equality – ε = 3.5; $\underline{n}$ = 9

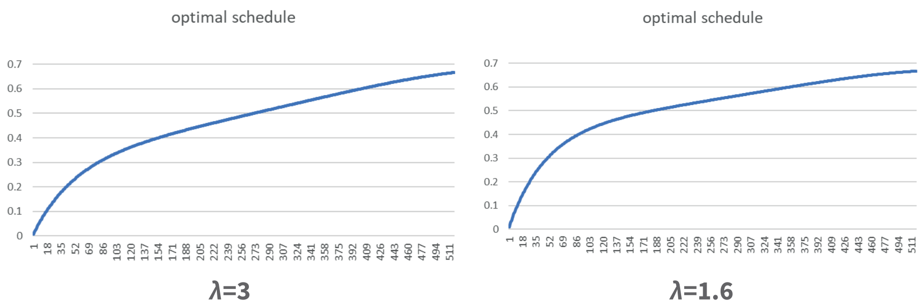
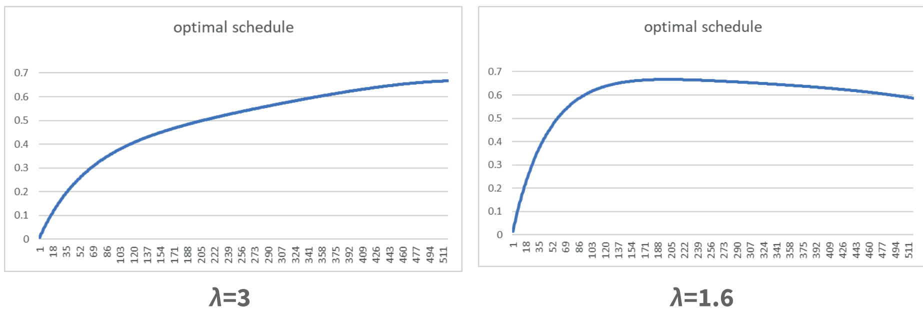


Figure 3b. Higher government subsidy: strong inequality – ε = 1.5; $\underline{n}$ = 12



I find that in these scenarios, average optimal tax rates are higher. This outcome is driven by the fact that a higher subsidy exacerbates economic inequality, leading to a rapid increase in optimal marginal tax rates. Regarding the marginal tax schedule, it is noteworthy that marginal tax rates rise much more gradually at the top. Unlike the steep increases observed in previous cases, the slope of the tax schedule flattens significantly in these instances. Notably, when the subsidy, s, is high, resulting in λ = 1.6, marginal tax rates even decline at the top, accompanied by rising average tax rates.

A plausible explanation for this pattern is the increased influence of the educational effect, as higher education participation grows substantially in scenarios with a high government subsidy (i.e., a low λ). Under such conditions, widespread enrollment in higher education becomes more likely, as seen in cases like South Korea.[12] This dynamic could result in optimal marginal tax rates that rise more gradually, with a potential decline at the top.

12 Participation at tertiary education in South Korea in 2023 stood at 54 percent (source: World Bank).

Summary and Conclusions

This paper examines the optimal income taxation schedule in an economy comprising skilled and unskilled workers. Skilled workers decide about their educational record by giving up to receive a wage in the period they study, with the purpose of increasing productivity in their future work by means of the learned material in higher education institutions.

The government plays a pivotal role in shaping higher education policies through subsidies and regulations, while individuals make decisions regarding education investment, labor supply, and consumption. Through redistributive policies, the government determines the optimal marginal income tax rates. By influencing education costs, government policies significantly affect both overall economic output and income inequality. These policies also shape the elasticity of demand for education, a critical factor in determining optimal tax rates.

Simulations using empirically calibrated parameters indicate that optimal income tax rates tend to increase with income. However, a declining slope for top marginal tax rates emerges when government subsidies and regulations for education are sufficiently generous, making education more accessible to a broader population. Under these circumstances, the demand for education becomes more elastic, leading to lower optimal marginal tax rates at the highest income levels.

A valuable extension of this analysis would involve an endogenous framework for income distribution that incorporates a middle class reliant on government support for education investment. Alternative education policies targeting the middle class could result in different steady-state income distributions and, consequently, distinct optimal income tax schedules.

Appendix A. Obtaining the F.O.C.

Following non-linear optimal income tax literature, I assume that utility is both additive and separable in leisure and consumption[13]:

(A.1) $$u = U(C) + V(1 - L)$$

where C is consumption, 1-L is leisure and U and V are respectively the utility of consumption and the utility of leisure. U and V are differentiable, concave and non-decreasing. The budget constraint at the individual level is:

13 Almost all papers that appear in the references are based on separable utility between consumption and leisure. Saez (2001) is an exception, although his simulations are also based on separable utility function.

$$C(w) = wL(w) - T[wL(w)] \tag{A.2}$$

where T symbolizes the income tax, which is defined on total income since the wage w and the supplied amount of labor L(w) are not observed by the government. The first order condition at the individual level is:

$$\frac{V_{(1-L)}}{U_C} \equiv -\frac{V_L}{U_C} = (1-\tau)w\,, \qquad \tau \equiv \frac{\partial T[wL(w)]}{\partial [wL(w)]} \tag{A.3}$$

where $V_{(1\text{-}L)}$ and U_C are the first derivatives of V and U, respectively.
Tthe government maximizes a social welfare function:

$$SW = \int_{w_L}^{\infty} G\{U[C(w)] + V[1-L(w)]\} f(w)dw \tag{A.4}$$

where w_L is the bottom of the positive and continuous distribution of skills which is denoted by F(w) and its respective density function is denoted by f(w). The budget constraint of the economy is:

$$\int_{w_L}^{\infty} C(w)f(w)dw = \int_{w_L}^{\infty} wL(w)f(w)dw \tag{A.5}$$

i.e., government intervention is purely redistributive.

The following is the Hamiltonian (H), which is composed by the social welfare utility function, the budget constraint of the economy and the differential equation for the state variable u given by a self-selection constraint:

$$H = \{G(u) - \gamma[C(w) - wL(w)]\}\frac{dF}{dw} + \varphi(w)V_L\frac{L}{w} \tag{A.6}$$

The control variable of this problem is L. γ is the multiplier of the budget constraint and λ is the multiplier of the self-selection constraint. The F.O.C. for a maximum are:

$$H_L = \left(w - \frac{dC}{dL}\Big|\bar{u}\right)\frac{dF}{dw} + \frac{\varphi(w)}{w}(V_L + LV_{LL}) = \gamma\left(w + \frac{V_L}{U_C}\right)\frac{dF}{dw} + \frac{\varphi(w)V_L}{w}\varepsilon = 0, \tag{A.7}$$
$$where\ \ \varepsilon = 1 + \frac{LV_{LL}}{V_L}$$

ε is a measure of labor supply elasticity. Note, that $\varepsilon = 1 + 1/\varepsilon^c$ in the linear utility of consumption case, where ε^c is the compensated elasticity of labor supply. In general, $\varepsilon=(1+\varepsilon^u)/\varepsilon^c$ where ε^u denotes the uncompensated elasticity of labor supply.

(A.8) $$H_u = \left(g - \gamma \frac{dC}{du}\right)\frac{dF}{dw} = \frac{d\varphi}{dw}, where\ g \equiv \frac{dG(u)}{du}$$

Dahan and Strawczynski (2014) show that combining 7 and 8 we get the following expression:

(A.9) $$\gamma(w + \frac{V_L}{U_C})f(w) + \frac{V_L \varepsilon \int_w^\infty \left[\frac{\gamma}{U_C(x)} - g(x)\right]\frac{dF}{dx}dx}{w} = 0$$

Introducing investment in education

To explain this feature let us assume that investment in higher education requires a minimal level X^*; above this level, there is a return on education at the labor market in the form of wage (w), which is obtained by individual i according to his/her learning capacity n and his/her investment in higher education X:

(A.10) $$w_i = n_i X_i, \quad X_i > X^*$$

The return to education depends both on the investment in education, X, and its return, n. I use the cost of education function, h[14], given in the following equation:

(A.11) $$h(X_i) = \Gamma_0(R) + \Gamma_1(R)\frac{{X_i}^{1+\lambda_i}}{1+\lambda_i}$$

Where λ_i represents the cost of education, which will be explained in equation 15; Γ_0 and Γ_1 represent parameters that are a function of R (government regulatory burden). Solving the optimum after assuming that the cost of education can be deducted from tax payments, it can be shown that the investment in higher education is[15]:

(A.12) $$X_i = \left(\frac{n_i}{\Gamma_1}\right)^{1/\lambda}$$

14 See Lafont (1994), page 220.

15 I assume that the government recognizes investment in education as an item that can be deducted from income tax payments; i.e., it is not subject to an income tax. This feature is quite usual in tax procedures in OECD countries.

Which implies:

(A.13) $$n_i = \Gamma_1 X_i^{\lambda_i}$$

i.e., at the optimum the return to education is affected by optimal investment, X, which is affected by the cost parameter, λ. The wage is:

(A.14) $$w_i = \widetilde{K} n_i^{\frac{1+\lambda_i}{\lambda_i}}, \quad X_i > X^*; \ \widetilde{K} = \left(\frac{1}{\Gamma_1}\right)^{1/\lambda}$$

Note that for n>1, the higher is λ, the more rigid is the demand for education for a given learning capacity. Equation 14 implies that the higher is the cost parameter λ, the lower is the wage. Thus, if the government is interested on large investment in higher education, it must assure a situation of a low λ, which will incentivate investment in higher education among the skilled.

Having said that, note that the government influences the cost of education in a partial manner, as characterized in equation 15 (see appendix A):

(A.15) $$\lambda = \lambda^1(s) + \lambda^2 + \lambda^3$$

Where λ^1 represents the direct cost of education (matriculation), and consequently is a function of s (government's subsidy to higher education); λ^2 represents the alternative cost of working; and λ^3 represents the financial cost of loans provided by banks for financing higher education. Thus, λ (in particular, the government-influenced part) will have a crucial impact, since it determines the education's elasticity of the demand by individuals that invest above the minimal education threshold – X^*.

Multiplying both the denominator and nominator of the RHS of equation 9 by [1-F(w)] and substituting w according to equation A.14, I obtain:

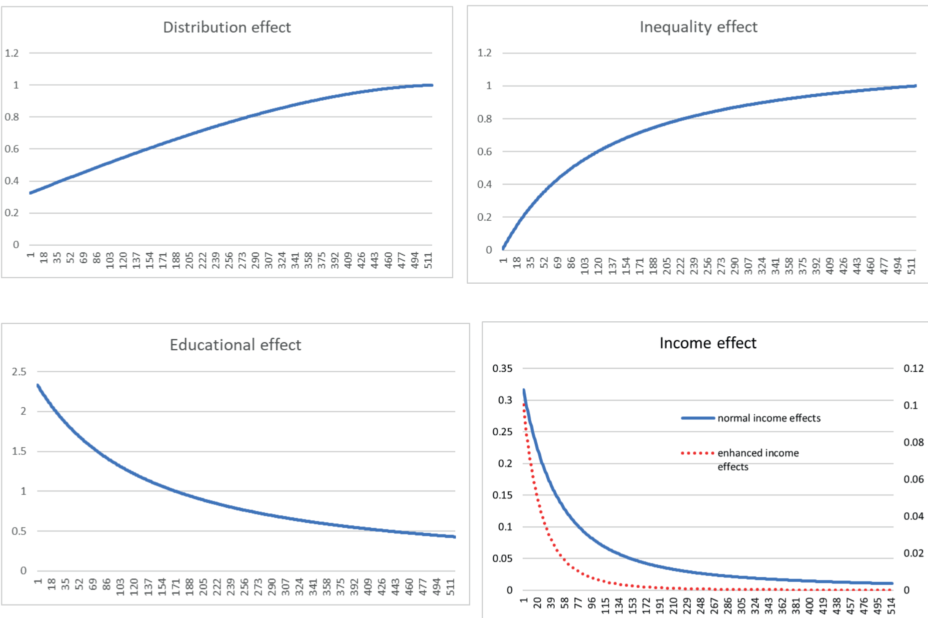
(A.16) $$\frac{\tau}{1-\tau} = \{\varepsilon[w(X)]\} U_C\big(w(X)\big) \left[\frac{\int_{w(X)}^{\infty}\left[\frac{\gamma}{U_C(x)} - g(x)\right] f(x)dx}{\gamma\big(1-F(w(X))\big)}\right] \left[\frac{\big(1-F(w(X))\big)}{f\big(w(X)\big)}\right] \frac{1}{\widetilde{K} n^{\frac{1+\lambda}{\lambda}}}$$

Equation (A.16) is the analytical expression to be used for the calculation of optimal tax rates using the distribution of skills (equation 7 in the text). This equation shows that the determination of the optimal shape of the tax schedule is given by the interaction of five effects. As explained in Dahan and Strawczynski (2014), the first term in the RHS of Equation (16) is a measure of labor supply elasticity: the higher the compensated elasticity of labor the lower the optimal marginal income tax rate. The second term, which is the marginal utility of consumption, affects the desired labor supply through its effect on labor supply. For high income levels, the marginal utility of consumption is low, and thus the incentive to work harder because of net income reduction disappears. The third effect is the "inequality aversion effect" which is increasing in w in the whole income range. The fourth effect is the "distribution effect": the higher the proportion of individuals above the income level relative to the proportion of individuals at this level, the less distortionary is the marginal tax rate, since for these individuals the marginal tax rate acts like a lump-sum tax. Thus, a higher ratio of (1-F) over f implies a higher optimal tax rate.

Appendix B – Effects that determine the optimal tax schedule

Since the efficiency effect is fixed, it affects the level of optimal tax rates but not schedule's shape; In this appendix I show the impact of other four effects when $\varepsilon = 2.5$ under normal income effects: two (the other two) of them drive rising (declining) marginal tax rates. The optimal shape is based on the interaction of all effects.

FIGURE B.1 – The shape of effects shown in Equation 7



Note that income effects under enhanced income effects are milder because of the impact of stronger income effects on labor supply. Since labor goes down as education return rises (opposite to what happens in the normal income effect case), this feature acts as a moderating force of income effects.

Appendix C – The cost of education

The following are the assumptions for different scenarios about the cost of education.

Table C.1 – Assumed cost at different scenarios

λ	$\lambda_s = 5$	
	S	U
$\lambda^1(s)$	2.5	2.5
λ^2	1.3	3.3
λ^3	1.2	1.9

The main component of the cost is the direct cost, λ^1, that is influenced by government subsidy, s. The alternative cost λ^2 is much lower for the skilled relatively to the unskilled, a gap that is much higher as a percent of wage. Concerning the discrimination in financial costs (λ^3) and as mentioned by Galor and Zeira (1993), it is a function of economic power, related to availability of collateral.

In Table C.2 I check the participation for different scenarios of government policy. For this purpose, I use an estimate based on Strawczynski (2014). Consistently with his finding on needed investment in education in OECD countries, For the given abilities distribution, I assume that in all scenarios is 11.35 percent of the minimal desired educational investment. It turns out that when $\lambda_S = 5$, tertiary participation approximates the one of OECD countries (40 percent); thus, I choose this parameter value as a benchmark scenario. The OECD figure is obtained as the average rate of participation in tertiary education by Australia, Belgium, Canada, Denmark, Finland, France, Germany, Iceland, Ireland, Israel, Japan, Korea, Luxembourg, Netherland, New Zealand, Norway, Spain, Sweeden, Switzerland, UK and US (Source: World Bank).

Table C.2 – Percent of Higher education participation as a function of government subsidies and regulation ($\underline{n}$ = 9)

	$\lambda_s = 1$	$\lambda_s = 2$	$\lambda_s = 3$	$\lambda_s = 5$	$\lambda_s = 10$	$\lambda_s = 12$
Participation under government policy (%)	93.2	86.1	78.6	40	11.4	10.5

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