András Simonovits

Family Allowance and Pension Dependent on the Number of Children: Optimality and Neutrality

Reaction to "The Inherent Faults of the State Pension System and the Main Direction of its Improvement" by József Banyár

SUMMARY: When it comes to the neutralisation of child-rearing burdens by the state, many experts support the introduction of pensions dependent on the number of children (or dependent on child-rearing) instead of family allowance (and tax allowance) (cf. Kovács ed., 2012). The critical review by Banyár (2019) joins this trend. Leaving external criticism aside, I assess Banyár's plan of reduced pension of childless people and the remaining family allowance in the simplest possible optimisation model. The relative child-consumption plays a key role. My main findings: a) in case of a critical relative value (when the total child consumption of the family equals the parents' consumption), both pension of the childless and family allowance are dispensable; b) in case of a lower child relative value, the reduced pension of the childless is positive, and family allowance is dispensable; c) in case of a higher relative value, the pension of the childless is zero, but family allowance is indispensable.¹

Keywords: pension system, family allowance, pension dependent on the number of children JEL codes: H55 DOI: https://doi.org/10.35551/PFQ_2020_1_3

Although *Banyár and Mészáros* (2003) have already examined the social security pension system dependent on the number of children in a multifaceted manner, the publication of *Botos and Botos* (2012) gave new impetus to the domestic debate around the issue (Kovács, 2012). (For the sake of brevity, we leave out the attribute 'social security', and we refer to the private pensions to be introduced for the childless as savings.) Numerous Hungarian economists (for example Banyár, 2012) argued that this delayed transfer system is more suitable to neutralise child-rearing expenses than family allowance without delay (as well as

E-mail address: simonovits.andras@krtk.mta.hu

family tax allowance and in-kind allowances). This difference in terms of delay manifests itself emphatically in the case of the tens of thousands of Hungarian workers moving permanently abroad at adult age. Although not every Hungarian economist accepts the arguments (for example, Mihályi, 2012; 2019; Németh, 2012; Simonovits, 2012), the approach is dominant for the time being, and it detrimentally overshadows other questions, which are at least as important as this one (the increasing polarisation of pensions within retirement years and between retirement years; the blending of the rigid and the flexible age limit for retirement).

Banyár's study (2019) has been published recently, and it provides a rich overview of the foreign and domestic literature on pensions dependent on the number of children. He is also in favour of the idea; he would deny childless people – subject to the obligation to pay contributions – pensions or would decrease their pensions to the minimum (as they can save the expenses of child-rearing), and he assigns a minimal role to family allowance. Surprisingly, the article includes neither formulae nor numerical recommendations for the reform.

In this paper, I will model Banyár's ideas in the simplest possible framework. I will set aside my own reservations, and I will only give an internal evaluation. For the sake of simplicity, there are no families and there are no genders (fathers have sons and mothers have daughters if you like), the fertility of those with children is identical (except for those in the Appendix), and that of childless people is, of course, 0. Furthermore, there is no growth, and the gross earnings of both types are identical. There is no inflation and interest, and labour supply and declarations of income are independent of the tax and contribution rates. I do not address some - otherwise very important - questions such as how to switch to such a system and how to operate it. I will examine only three minimalist models; they do not include any hypotheses which would improve the model family only in a quantitative manner but would leave their inherent characteristics unchanged. I specifically point out that contrary to, for example, Simonovits (2014) we consider fertility independent of family allowances.

There is neither family allowance nor pension in Model 1; everyone takes care of their old age themselves. In the optimum the young and old age consumption of both types are equal. We suppose that those with children spend on their children's consumption in proportion of their own adult consumption; therefore, the optimal consumption of people without children is greater than the (adult) consumption of those with children. (We call the proportionality multiplier the 'consumption specific value of children'.)

In Model 2 people with children receive family allowance in the child-rearing period (including family tax allowance and other allowances relating to child-rearing allocated free of charge), and each pensioner receives the same pension. By appropriately selecting the family allowance and the contribution, neutrality can be ensured in addition to the optimum: adult consumption is independent of type. The two terms in the title are important because without them, the goal and behaviour of the individuals and the government would be unidentified. By the way, in the traditional approach individuals maximise their own lifetime utility function, whereas the government does the same to the social welfare function (cf. Simonovits, 2014).

In Model 3 we analyse Banyár's ideas about pensions maximally dependent on the number of children bare and appropriately modified. (The current French pension reform would increase the pension of mothers by a 'mere' 5 percent per child, which would presumably not satisfy the domestic supporters of the system.) First and foremost, a classification

must be introduced: the child specific value is lower/higher than the critical value. (In our specification, this would precisely mean that the total consumption of children is lower/higher than that of the parents.) Oddly enough, in addition to neutrality and optimisation this classification is also missing from Banyár's paper, that is why no emphasis is given to the fact that in case of a lower specific value, childless people also receive pensions - even if a reduced amount - in the neutral optimum. In case of a higher specific value, however, besides the abolition of the pension of the childless the introduction of family allowance would be unavoidable in this system. Finally, we will deduct the continuum between the two extremities.

The realism of the model family can be enhanced by taking into account that a) the length of the time period spent in retirement and that of child-rearing is are about the half of the time period spent working, b) the number of children varies in families with children and c) gross earnings are heterogeneous. We briefly outline case a) and case b), and in case c), the pensions and the savings would be in proportion with the earnings, but family allowance would be replaced by the earningsdependent family tax allowance in case of an appropriately high maximum. The inclusion of the dynamics would present the real difficulty. In order to illustrate the scales, we illustrate our results through numeric examples.

The structure of the paper is as follows. After the introduction, Section 2 examines the pure market system without transfers. Section 3 formulates the neutral and optimal transfer rules. Section 4 outlines the most natural implementation of this system: family allowance and uniform pension. Section 5 presents the combination of maximum pension dependent on the number of children and minimal family allowance. Section 6 presents the numeric results of the generalisation mentioned in point a) for the critical relative value. Section 7 draws the conclusions. The paper is closed by an Appendix.

LIFE CYCLES WITH PURE MARKET

In this section, there is no state that would modify the consumption of families with varying numbers of children by providing family allowance or differentiated pension. Here and hereinafter most of the variables are arbitrary real numbers.

Two types coexist: H has *n* children (not necessarily an integer), L has no children, their weights in the population is are respectively f > 0, 1 - f > 0. Population is stationary: fn = 1, that is, n = 1/f > 1. Both types earn the same amount in the first period (25–30 years), and the consumption of each child is φ times the adult's: $0 < \varphi \le 1$ is the relative consumption of the child. The consumption of all the children of one parent is φn . In this simple family model, there is no interest and population growth; therefore, we can safely consider savings $s_{\rm L}$ and $s_{\rm H}$ as differentiated pension contribution too, which is the pension itself, at the same time.

The following equations apply:

Young age (adult) consumption

$$c_{\rm L} = 1 - s_{\rm L}$$
 and $c_{\rm H} = \frac{1 - s_{\rm H}}{1 + \varphi n}$. (1)

Old age consumption:

$$d_{\rm L} = s_{\rm L}$$
 and $d_{\rm H} = s_{\rm H}$. (2)

As there is no interest rate, there is no discounting and the existence of the child does not make the parents happy by itself, in an optimal case young age and old age consumption are equal:

$$c_{L} = d_{L}$$
 and $c_{H} = d_{H}$. (3)

Theorem 1

In a pure market system the optimal savings of the two types are respectively

$$s_{\rm L}^{\rm o} = \frac{1}{2}$$
 and $s_{\rm H}^{\rm o} = \frac{1}{2 + \varphi n}$ (4)

that is, the optimal consumption path of childless people moves above those with children:

$$c_{\rm L}^{\rm o} = d_{\rm L}^{\rm o} = \frac{1}{2} > c_{\rm H}^{\rm o} = d_{\rm H}^{\rm o} = \frac{1}{2 + \varphi n}$$
 (5)

PROOF. Let's start with type L. Substituting it into (1L)–(2L)-t (3L) results in $1 - s_L = s_L$, which is followed by (4L). Let's continue with type H. Substituting it into (1H)–(2H)-t (3H) results in $1 - s_H = (1 + \varphi n)s_H$, which is followed by (4H). Substituting (4) into (3) results in (5).

We illustrate our results through a numeric example. f = 1/2, that is, n = 2. Table 1 shows the market optimum for 3 specific parameter values – none is neutral. The higher the specific value, the more the adult consumption of those with children falls short of that of childless people.

NEUTRAL TRANSFER SYSTEMS

Alongside the market system, the modern state operates transfers dependent on the number

of children and other (for example earningsdependent) transfers. Let the transfer given by type L be $t_{\rm L} \ge 0$ and the transfer received by type H be $t_{\rm H} \ge 0$, that is the lifetimebalance

$$c_{\rm L} + d_{\rm L} = 1 - t_{\rm L}$$
 and $c_{\rm H}(1 + \varphi n) + d_{\rm H} = 1 + t_{\rm H}$.

The sum of the given transfers equals that of the transfers received:

$$(1-f)t_{\rm L}=ft_{\rm H}$$

We will see that both family allowance and pension depending on the number of children divert individual lifetime consumption from lifetime earnings in favour of people with children. As average family consumption equals the average earnings, the following is true here as well

$$(1-f)(c_{\rm L}+d_{\rm I})+f[(1+\varphi n)c_{\rm H}+d_{\rm H}]=1.$$
 (6)

The simplest way to express the equalising endeavours of family policy is neutrality. We define a transfer system *neutral* if the adult consumption pair is independent of the number of children:

$$c_{\rm L} = d_{\rm L}$$
 and $c_{\rm H} = d_{\rm H}$. (7)

Table 1

Children consumption	Without children	With children	Without children	With children	
specific value	savings		adult consumption		
φ	S ⁰ _L	<i>S</i> ⁰ _Н	$\boldsymbol{\mathcal{C}}_{L}^{0}=\boldsymbol{\mathcal{A}}_{L}^{0}$	$C_{\rm H}^{\rm o} = d_{\rm H}^{\rm o}$	
0.3	0.500	0.385	0.500	0.385	
0.5	0.500	0.333	0.500	0.333	
0.7	0.500	0.294	0.500	0.294	

OPTIMAL MARKET CONSUMPTION PATHS

Source: own editing

(We will see that there are various neutral transfer systems.)

In this section, we examine the potential neutral optimal transfer systems without the specification. In accordance with (3), the optimal young and old age consumption value of both types are equal. In a neutral case, however, the four values are equal, and their common value is e^* .

Theorem 2

a) Common adult consumption value in an arbitrary neutral optimal transfer system

$$e^* = \frac{1}{2+\varphi}$$
 (8)

b) The payment of a childless person and the payment of a person with children are respectively

$$t_{I}^{*} = \varphi e^{*}$$
 and $t_{H}^{*} = (n-1)\varphi e^{*}$:

Note. We can observe that under our hypothesis relating to stationary population – one young adult raises 1 child on average – the transfer paid by a childless person is precisely the consumption of one child, and the transfer received by a person raising n children is precisely the consumption of n-1 child. This can also be expressed by saying that the expenses of child-rearing are equally distributed between those without children and those with children.

PROOF. *a*) The lifetime consumption of type L is 2*e*, whereas that of type H (the consumption of the children being included) is $2e + \varphi ne$. In accordance with (6),

$$(1-f)(2e) + f(2e + \varphi ne) = 1,$$

which results in (8) by taking into account fn = 1.

b) in accordance with (8), due to the transfer given by a childless person

$$t_{\rm L}^* = 1 - 2e^* = \frac{2 + \varphi - 2}{2 + \varphi} = \frac{\varphi}{2 + \varphi}$$

and the transfer received by a person with children

$$t_{\rm H}^* = (1-f)t_{\rm L}^*/f = (n-1)t_{\rm L}^*$$
.

FAMILY ALLOWANCE AND PENSION INDEPENDENT OF THE NUMBER OF CHILDREN

In this section, we model the pension system currently in place in Hungary: alongside family allowance (and other allowances dependent on the number of children), the pension is independent of the number of children.

Both types of workers pay τ pension contribution. (In fact, in Hungary, those with children have been able to deduct the family tax allowance from the employee's pension and health care contributions since 2014! If this correction were applied in the employee's net earnings used in the calculation of pension, the Hungarian pension system would be dependent on the number of children – but only in the case of those with lower earnings.) Furthermore, type L pays special tax θ , and type H receives $\theta(1-f)/f = \theta(n-1)$ family allowance. (In fact, everyone pays uniform special tax, but only those with children receive transfers from these - this was netted in the model, though.) Both types continue to live off pension b for a unit of time. The following equations apply:

Young age (adult) consumption:

$$c_{\rm L} = 1 - \tau - \theta$$
 and $c_{\rm H} = \frac{1 - \tau + \theta(n-1)}{1 + \varphi n}$ (9)

Old age consumption:
$$d_{\rm L} = d_{\rm H} = b.$$
 (10)

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The pension system is also self-financing:

$$b=\tau.$$
 (11)

In the individual optimum, young and old age consumption are again equal for both types, and optimal neutrality can be achieved without savings by the appropriate choice of the contribution and the special tax.

Theorem 3

In case of traditional family allowance and uniform pension, the optimal neutral consumption path is given by the following contribution and special tax:

$$\tau^* = \frac{1}{2+\varphi} = e^* \quad \text{and} \quad \theta^* = \frac{\varphi}{2+\varphi} = t_{\rm L}^*. \tag{12}$$

PROOF. In the neutral optimum, in accordance with (10)–(11), the contribution equals adult consumption: $\tau^* = e^*$; and the optimal special tax equals the consumption of one child: $\theta^* = \varphi e^*$.

At the end of the section, we will illustrate our model with numbers again in *Table 2*. As the specific value is increasing, the pension contribution is decreasing while the special tax is increasing: in case $\varphi = 1$ the two quantities are identical.

PENSION DEPENDENT ON THE NUMBER OF CHILDREN AND FAMILY ALLOWANCE

Now, we are getting to the central question of our article: how can family allowance be reduced to the minimum by the maximum extension of pension dependent on the number of children while maintaining the neutral optimum? Furthermore, we assume that the pension of H is so high that he/she does not need to save for old age: $s_{H}^{*} = 0$, while the pension of L is so low that he/she has to retire with large savings to supplement his/ her reduced pension to an appropriate level [(17)].

Young age (adult) consumption

$$c_{\rm L} = 1 - \tau - s_{\rm L} - \theta$$
 and $c_{\rm H} = \frac{1 - \tau + \theta(n-1)}{1 + \varphi n}$. (13)

Old age consumption

$$d_{\rm L} = b_{\rm L} + s_{\rm L} \quad and \quad d_{\rm H} = b_{\rm H}. \tag{14}$$

Table 2

NEUTRAL OPTIMAL CONSUMPTION PATHS: FAMILY ALLOWANCE AND PENSION CONTRIBUTION

Children	Pension	Special	Without children	With children
consumption specific value	contribution	tax	adult consumption	
φ	τ*	θ*	$\boldsymbol{C}_{L}^{\star} = \boldsymbol{d}_{L}^{\star}$	$\boldsymbol{C}_{H}^{\star} = \boldsymbol{d}_{H}^{\star}$
0.3	0.435	0.130	0.435	0.435
0.5	0.400	0.200	0.400	0.400
0.7	0.370	0.259	0.370	0.370

Source: own editing

Pension contribution covers the pensions:

$$\tau = f_{\rm L} b_{\rm L} + f_{\rm H} b_{\rm H}. \tag{15}$$

The result of the case selection

Theorem 4

In case of maximum pension dependent on the number of children, in function of the parameter values 3 different neutral optimums may exist.

If $\varphi n = 1$, there is neither family allowance: $\theta^* = 0$ nor pension of the childless: $b_1^* = 0$.

If $\varphi n < 1$, there is no family allowance: $\theta^* = 0$, but there is a reduced pension of the childless: $0 < b_L^* = (1 - \varphi n) b_H^* < b_H^* = e^*$.

If $\varphi n > 1$, there is family allowance: $\theta^* > 0$, but pension of the childless does not exist: $b_L^* = 0$.

NOTE. The value of the appropriate contribution, family allowance and savings are shown in the proof. The most interesting result is the formula of the reduced pension of the childless: $b_{\rm L}^* = (1 - \varphi n) b_{\rm H}^*$, that is, the pension of the childless decreases in proportion to the expenses of child-rearing.

PROOF. We suppose that there is a neutral optimum, and based on Theorem 2 all the adult consumptions of equation (13)-(14) are

equal to e^* . Let's rewrite the neutral optimum of (13)–(14) to get a slightly simpler form:

$$e^* = 1 - \tau^* - s_L^* - \theta^*$$
 and
 $e^* = 1 - \tau^* + (n-1)\theta^* - \varphi n e^*.$ (13')

$$e^{*} = b_{\rm L}^* + s_{\rm L}^*$$
 and $e^{*} = b_{\rm H}^*$. (14')

by comparing the two equations of (13°): $s_{\rm L}^* = n(\varphi e^* - \theta^*).$ (16) by comparing the two equations of (14°):

$$b_{\rm L}^* + s_{\rm L}^* = b_{\rm H}^*.$$
(17)

IN WORDS: (i) the savings of a childless person = number of children × (consumption of the child – special tax); (ii) pension of the childless person + the savings of the childless person = the pension of the person with children.

ad a) In case $\varphi n = 1$ we can try $\theta^* = 0 = b_{\rm L}^*$ selection, $s_{\rm L}^* = e^* = b_{\rm H}^*$.

Then, in accordance with (14), $\tau^* = e^*/n$.

ad b) In case $\varphi n < 1$ $\theta^* = 0$ is appropriate, in accordance with (16) $s_L^* = \varphi n e^*$, that is in accordance with (17), $b_L^* = (1 - \varphi n) e^* > 0$. In accordance with (14), $\tau^* = [1 - (n-1)\varphi] e^*$.

ad c) in case $\varphi n > 1$ the pension of the childless cannot be reduced further: $b_{\rm L}^* = 0$,

Table 3

OPTIMAL CONSUMPTION PATHS: PENSION MAXIMALLY DEPENDENT ON THE NUMBER OF CHILDREN

Children consumption specific value	Pension contribution	Special tax	Additional savings	Without children With childr adult consumption	
φ	τ*	θ*	s,⁺	$\boldsymbol{C}_{L}^{*} = \boldsymbol{d}_{L}^{*}$	$\boldsymbol{c}_{H}^{\star} = \boldsymbol{d}_{H}^{\star}$
0.3	0.304	0.000	0.261	0.435	0.435
0.5	0.200	0.000	0.400	0.400	0.400
0.7	0.185	0.074	0.370	0.370	0.370

Source: own editing

so in accordance with (17) $s_{\rm L}^* = e^*$, $\tau^* = e^*/n$ known from *a*) remains, but this time in accordance with (13-1), family allowance is $\theta^* = 1 - n^{-1}e^* - 2e^*$.

Finally, we illustrate Theorem 4 in *Table* 3 through a numerical example. We show the case of a lower specific value in row 1, where family allowance is not needed, but the reduced pension of the childless is. Row 2 precisely includes the critical case, and in row 3 we present the case of a higher specific value, where the additional burdens of those with children are neutralised by the family allowance brought back, in addition to the anulled childless pension.

At this stage, it is worth mentioning that the common example of Botos–Botos (2012) is well approached by the $0 < b_{\rm L}^* = (1 - \varphi n) b_{\rm H}^*$ $< b_{\rm H}^* = e^*$ formula of Point *b*) of Theorem No: 4 if we calculate with $\varphi = 2/7$. Then $b_{\rm H}/b_{\rm H} = 1, 4/0, 6 = 7/3$.

Finally, we show that the transition between the minimally and maximally differentiated pensions is continuous. We index the variables and parameters of the undifferentiated and maximally differentiated system with N and D, and we leave out the asterisk everywhere (except for e).

Theorem 5

The pension contribution realising the neutral optimum supplements the special tax covering the family allowance as follows:

$$\tau(\theta) = [1 - (n - 1)\varphi]e^{\star} + (n - 1)\theta,$$

where
$$0 \le \theta^{D} \le \theta \le \theta^{N},$$
 (18)

where θ^{D} and θ^{N} are the functions of φ specific value not detailed herein.

NOTE: The special tax term is the family allowance itself. The higher the family allowance, to a lesser extent the pension of the childless must be reduced. Therefore, the more the contribution increases.

PROOF. According to (16)–(17)

$$b_{\rm L} = b_{\rm H} - s_{\rm L} = e^* - n(\varphi e^* - \theta) =$$

(1-n\varphi)e^* + n\vartheta. (19)

Let's substitute (19) into (15):

$$\tau = (1 - f_{\rm H})[(1 - n\varphi)e^* + n\theta] + f_{\rm H}e^*.$$

Using that
$$f_{\rm H} = n^{-1}$$
, results in (18).

EXAMPLE: For the sake of simplicity, we separately examine the critical specific value: $\varphi = 1/n = f_{\rm H}$. Then the conversion function

$$\tau(\theta) = \frac{e^*}{n} + (n-1)\theta$$
, where $\theta^{N} = \theta$ és $\theta^{D} = \varphi e^*$.

With the data of our previous numerical example: $\tau(\theta) = 0, 2 + \theta$.

SHORTER CHILD-REARING PERIOD AND RETIREMENT

In my previous works, I often used a simple modification which allows us to get rid of the rough assumption typical of two-generation static models: the length of the period of child-rearing and that of retirement equal the active period. We simply introduce a positive number $\mu \in (0.1]$, which reduces the length of the active period to the supposedly common length of child-rearing and retirement. I prepared this paper with this generalisation as well, but due to is complexity we only write down the simplest connections (by referring to the modification by using commas).

 $c_{\rm L} + \mu d_{\rm L} = 1 - t_{\rm L}$ and $c_{\rm H} (1 + \mu \varphi n) + \mu d_{\rm H} = 1 + t_{\rm H}$.

NEUTRAL OPTIMAL CONSUMPTION PATHS: SHORTER PERIODS						
Model	Pension contribution	Special tax	L	Н	L	н
			savings		adult consumption	
	τ*	$ heta^*$	s _L	S _H	$\boldsymbol{C}_{L}^{\star} = \boldsymbol{d}_{L}^{\star}$	$\boldsymbol{C}_{H}^{*} = \boldsymbol{d}_{H}^{*}$
Model 1	0.000	0.000	0.333	0.250	0.667	0.500
Model 2	0.286	0.143	0.000	0.000	0.571	0.571
Model 3	0.143	0.000	0.286	0.000	0.571	0.571

Source: own editing

Considering the average:

$$(1-f)(c_{\rm I}+\mu d_{\rm I})+f[(1+\mu \varphi n)c_{\rm H}+\mu d_{\rm H}]=1. \tag{6'}$$

Theorem 2'

The common adult consumption value in an arbitrary neutral optimal transfer system

$$e^* = \frac{1}{1 + \mu(\varphi + 1)}$$
 (8')

As comparison with the idealised case we write down the results of a market and two neutral models in Table 4 for the critical specific value ($\varphi = 1/2$). The pair of savings in (market) Model 1 decreased from (0.5; 0.333) to (0.333; 0.25), and the pair of consumption goes up from (0.5; 0.333) to (0.667; 0.5). The contribution rate of 0.4 of Model No. 2 (uniform pension, family allowance) decreases to 0.286 in a more realistic parametering whereas the adult consumption increases from 0.4 to 0.571. Similarly, the contribution rate of 0.2 of Model 3 (zero pension of the childless and zero family allowance) decreases to 0.143 in a more realistic parameterization (whereas adult consumption increases from 0.4 to 0.571).

CONCLUSIONS

In this short paper, we commented on the article written by Banyár (2019). Firstly, we presented what burdens are placed by the pure market solution on those with children. Secondly, we analysed the harmonious joint operation of family allowance and the uniform pension system. Thirdly, we modelled Banyár's pension system maximally dependent on the number of children, and we pointed out that there is a complicated quantitative connection between the optimal neutral family allowance and pension contribution:

a) if the child's relative consumption is critical, neither family allowance nor the pension of the childless is needed,

b) if the relative value is lower, family allowance is dispensable; but the pension of the childless can be abolished only partly,

c) if the relative value is higher, the pension of the childless can be reduced to 0, but family allowance is still needed.

Naturally, the quantitative findings formulated herein are sensitive to the determination of the model family parameters. If we considered that the time spent both child-rearing and in retirement are shorter than the active period, or

Table 4

pensioners consume less than workers, and, furthermore, positive fertility is were also varied, or if we formulated neutrality more vaguely or more strictly than we do herein, some of the quantitative relationships would probably change. However, our models are only for 'playing' and their purpose is solely to polish the mind.

Appendix

A MORE REALISTIC DISTRIBUTION OF NUMBER OF CHILDREN

In this *Appendix*, we address the hypothesis that all parents with children raise the same number of children. Let i = 0, 1, 2, ..., I be the number of children, and let c_i, d_i and f_i be the adult consumption of people with children i at a young and at an old age, and their weight in the population. Following Section 6, the length of the period spent child-rearing and in retirement is $\mu \le 1$.

For the sake of brevity we only generalise the neutral optimal transfer (Sections 3 and 6). THEOREM A.1

The common adult consumption in the neutral optimum

$$e^* = \frac{1}{1 + \mu(\varphi + 1)}$$
 (A.1)

NOTE. By comparing (8') and (A.1) we can see that the hypothesis of binary fertility reflects at least the average well.

PROOF. Lifetime consumption of type *i* is $c_i(1 + \mu \varphi i) + \mu d_i$. The average lifetime consumption in the balanced transfer system is 1:

$$\sum_{i=1}^{I} f_i [c_i (1 + \mu \varphi i) + \mu d_i] = 1.$$
(A.2)

By substituting into (A.2) the optimum condition: $c_i = d_i$, and the neutrality condition: $c_0 = \cdots = c_i = e$, and the weight sum: $\sum_{i=1}^{I} f_i = 1$ we receive $e[1 + \mu(\varphi + 1)] = 1$, that is, (A.1).

Note

¹ I would like to express my gratitude to József Banyár for his help provided while the article criticising him was written. It is especially important to highlight here that this does not mean that my debate partner agrees with all (or even a single one of) the statements expressed in the article. I must also thank Edina Berlinger for her valuable remarks and the anonymous referee for their constructive criticism and for identifying a miscalculation.

References

BANYÁR, J. (2012). Gyermeknevelés és nyugdíj – összekapcsolható vagy sem? (Child-rearing and pension – Can they be linked or not?) Kovács, E. (ed.) pp. 61–89 BANYÁR, J. (2019). Az állami nyugdíjrendszer születési hibái és javításának főiránya. (The Inherent Faults of the State Pension System and the Main Direction of its Improvement.) Pénzügyi Szemle/ Public Finance Quarterly, No. 4, pp. 511–525 https://doi.org/10.35551/PFQ_2019_4_4

BANYÁR, J., MÉSZÁROS, J. (2003). Egy lehetséges és kívánatos nyugdíjrendszer. (A possible and desirable pension system.) Gondolat Kiadó, Bp.

BOTOS, K., BOTOS, J. (2012). Nyugdijrendszerünk jövője. (The future of our pension system.) Kovács, E. (ed.) pp. 15–22

Kovács, E. (ed.) (2012). Nyugdíjrendszer és gyermekvállalás. (Pension system and having children.) Gondolat Kiadó, Budapest

MIHÁLYI, P. (2012). Húsz érv a nyugdíj és a gyermekszám összekapcsolása ellen. (Twenty arguments against linking the amount of pensions to the number of children). Kovács, E. (ed.), pp. 144–162 MIHÁLYI, P. (2019). A gyermekvállalás határhasznai és határköltségei. (Marginal utilities and marginal costs of having children.) Pénzügyi Szemle/ Public Finance Quarterly, No. 4, pp. 526–541, https://doi.org/10.35551/PFQ_2019_4_5

NÉMETH, GY. (2012). Gyermekvállalás – lehet-e gazdaságilag racionális döntés? (Having children – Can it be an economically rational decision?) Kovács, E. (ed.), pp. 23–38

SIMONOVITS, A. (2012). Gyermekszám és nyugdíj: kritika. (Number of children and pension: criticism.) Kovács, E. (ed.) pp. 163–169

SIMONOVITS, A. (2014). Gyermektámogatás, nyugdíj és endogén/heterogén termékenység – egy modell. (Childcare allowances, pension and endogenous/ heterogeneous fertility – one model.) Közgazdasági Szemle (Economic Review), 61, pp. 672–692